Number Sense

14 questions	12 questions	17 questions	17 questions	8 questions	12 questions
Number Sense	**Statistics, Data Analysis, Probability**	**Algebra and Functions**	**Measurement and Geometry**	**Math Reasoning**	**Algebra I**

CAHSEE contains 14 Number Sense items. Students' understanding of fractions, decimals, percents, and integers and their relationship to each other and to the other disciplines of mathematics is an essential component of their mathematics learning. CAHSEE questions in the Number Sense strand require students to demonstrate a foundational understanding of numbers and ways they are represented.

Students will be asked to:

- solve problems with fractions, decimals, and percents.

- compare and order numbers.

- demonstrate an understanding of percents, including those less than 1 and greater than 100.

- understand and meaningfully interpret large and small numbers in scientific notation.

- use specific characteristics of numbers, such as multiples, factors, and primes.

- use and represent integers as the basis for the comparison of quantities.

Essential to success in this CAHSEE strand is the student's understanding of the mathematical operations and the ways they are related to each other. This understanding includes:

- the meaning of arithmetic operations with fractions, decimals, and integers.

- the associative and commutative properties of addition and multiplication.

- the distributive property of multiplication over addition.

- the understanding and use of inverse relationships of addition and subtraction, multiplication, and division.

- finding square roots, squaring numbers, and using the inverse relationship between them.

Students also should possess computational fluency. They should be able to select appropriate methods and tools for computing with fractions and decimals; perform mental arithmetic; use algorithms for computing with fractions, decimals, and integers; use strategies for estimation and for judging the reasonableness of results; and be able to analyze and explain methods for solving problems with proportions.

The ten California academic content standards covered by the CAHSEE Number Sense strand are discussed in the following pages.

Strand **Number Sense (NS)**

Standard **7NS1.1**
Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10) with approximate numbers using scientific notation.

1 test question

The radius of the earth's orbit is 150,000,000,000 meters. What is this number in scientific notation?

A $\quad 1.5 \times 10^{-11}$

B $\quad 1.5 \times 10^{11}$

C $\quad 15 \times 10^{10}$

D $\quad 150 \times 10^{9}$

M00213

Scientific notation is required knowledge in science and engineering because many numbers are either so large or so close to zero that there is no other convenient way to write them. CAHSEE questions in this standard require students to demonstrate understanding of the basic concepts of scientific notation using approximations of very large and very small numbers. Test questions may also involve the translation of approximate numbers into scientific notation, the comparison of numbers in scientific notation with either positive or negative exponents, and the understanding of the relative size of two numbers in scientific notation.

Sample Test Question
The correct answer is choice B. Students should recognize that the place-value distance from the 1 (highest place value, 100 billion) to the decimal is 11 digits and that this value provides an appropriate representation of the equivalence as 10^{11}, also equivalent to $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$. Students also should know that expressions in scientific notation may include a multiplier, between 1 and 10, along with an exponential value of 10. While exact powers of 10 are expressed without a multiplier (e.g., 10^{11}), numbers such as 150,000,000,000 require a multiplier along with the equivalent power of 10. A typical method of finding the multiplier and the exponent is to count the number of decimal places the decimal must move to create a number between 1 and 10. In the example, the decimal point is moved 11 places to the left to get 1.5 for the multiplier and $+11$ for the exponent.

Analysis of Distractors
Distractor A: expressed the power of 10 as $\dfrac{1}{10^{11}}$ (students are often unsure of the direction the decimal point should move)

B: correct answer

Distractor C: equivalent in number to 150,000,000,000, but not in scientific notation

Distractor D: equivalent in number to 150,000,000,000, but not in scientific notation

Strand	**Number Sense (NS)**

Standard   **7NS1.2**
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

3 test questions

John uses $\frac{2}{3}$ of a cup of oats per serving to make oatmeal. How many cups of oats does he need to make 6 servings?

A $2\frac{2}{3}$

B   4

C $5\frac{1}{3}$

D   9

M23015

All students should understand the basic arithmetic functions involving rational numbers in all forms and be comfortable performing calculations with positive and negative numbers. CAHSEE questions in this standard require students to demonstrate computational fluency with rational numbers and an understanding of the relationships between these types of numbers. These skills are fundamental to achievement of the California academic content standards in mathematics.

Sample Test Question
The correct answer is choice B. Students should understand that multiplying the initial amount of oats by the 6 servings yields the easiest calculation.

Analysis of Distractors

Distractor A: added the two numbers and made a mistake in the calculation $\left(\frac{2}{3}+\frac{6}{1}=\frac{8}{3}=2\frac{2}{3}\right)$

           B: correct answer

Distractor C: subtracted $\frac{2}{3}$ from the 6 servings

Distractor D: divided 6 by $\frac{2}{3}$

Strand	**Number Sense (NS)**

Standard **7NS1.3**
Convert fractions to decimals and percents and use these representations in estimations, computations and applications.

2 test questions

> **Some students attend school 180 of the 365 days in a year. About what part of the year do they attend school?**
>
> **A**   18%
>
> **B**   50%
>
> **C**   75%
>
> **D**   180%
>
> M00047

Students using mathematics in their daily lives will need to know how to convert decimals to fractions to percents with ease. The recognition of equivalent forms is essential for student fluency with numbers. CAHSEE questions in this standard require students to demonstrate the ability to find equivalent values and representations for numbers as well as to find and compute values using fractions, decimals, and percents. Students must also be able to use these various representations for estimating and performing computations in mathematical applications.

Sample Test Question
The correct answer is choice B. Students should recognize that the correct value is found by dividing the part (180) by the whole (365), giving $180 \div 365 \approx 0.493$. The question also requires conversion of 0.493 to a percent by multiplying by $100 (0.493 = 49.3\%)$. Finally, students should recognize that the phrase "about what part" in the stem calls for a rounded percent rather than an exact value and that $49.3\% \approx 50\%$. It is to be expected that many students will use mental estimation skills, rather than computation, to determine that 180 is about half of 365.

Analysis of Distractors

Distractor A: converted 180 to 18%

      B: correct answer

Distractor C: incorrect computation of the decimal equivalent of $\dfrac{180}{365}$ or a possible guess with apparent plausibility

Distractor D: converted 180 to 180%

Strand	**Number Sense (NS)**

Standard   **7NS1.6**
Calculate the percentage of increases and decreases of a quantity.

1 test question

The cost of an afternoon movie ticket last year was $4.00. This year an afternoon movie ticket costs $5.00. What is the percent increase of the ticket from last year to this year?

A   10%

B   20%

C   25%

D   40%

M02158

CAHSEE questions in this standard require students to demonstrate understanding of percent increase and decrease, which is a fundamental tool in analyzing numerical information. For example, a price change of one dollar can be very meaningful in terms of buying a loaf of bread and inconsequential in terms of buying a car. Students should understand that percent change clarifies the impact of this kind of change, and they should be able to calculate the change easily. Standard 7NS1.6 will also be assessed with questions that require students to find the percent decrease. Finding the quantity that results from a given percentage increase or decrease is assessed in standard 7NS1.7.

Sample Test Question
The correct answer is choice C. Students should understand that finding the percent increase or decrease of a quantity requires first finding the difference between the initial value and the final value. In the sample question, the difference is $1.00. Then to find the percent increase, students must know to compare the difference to the initial cost by using division: $\dfrac{\$1.00}{\$4.00}$. Finally, the resulting decimal must be converted to its equivalent percent: $1 \div 4 = 0.25 = 25\%$.

Analysis of Distractors

Distractor A: took the difference between the two ticket prices and incorrectly converted it to a percentage

Distractor B: multiplied both given values and converted to a percentage

        C: correct answer

Distractor D: converted the original ticket price to a percentage

Strand **Number Sense (NS)** Standard **7NS1.7** **Solve problems that involve discounts, markups, commissions, and profit, and compute simple and compound interest.** **2 test questions**	**Sally puts $200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in three years?** **A** $16.00 **B** $24.00 **C** $48.00 **D** $160.00 M02119

CAHSEE questions in this standard require students to solve a variety of problems involving percents. Both consumers and people working in business need to understand the mathematical meaning of common business terms such as commission, tips, profit, and loss, as well as how to make interest and tax computations. Solving problems of these types is one of the most important skills students need as they become adults. Understanding these concepts and their applications can assist students in managing their money and other resources. This standard will also be assessed with test questions that require students to find simple and compound interest, as well as discounts, markups, taxes, tips, and commissions. A maximum of three iterations is used for questions that involve calculating compound interest. The iterations include the initial multiplication of principal by interest rate.

Sample Test Question
The correct answer is choice C. Students should recognize that simple interest is calculated by multiplying the principal by the annual rate and then multiplying by the time. In the sample question, the principal is $200, the rate is 8%, and the time is 3 years. To calculate correctly, students are also required to convert 8% to its decimal equivalent $($200 \cdot 0.08 \cdot 3 = $48)$.

Analysis of Distractors

Distractor A: one year interest; multiplied the principal and interest rate only

Distractor B: multiplied the rate times the number of years only

C: correct answer

Distractor D: converted the interest rate to a decimal incorrectly $($200 \cdot 0.8)$, and did not multiply by the number of years

Strand	**Number Sense (NS)**

Standard **7NS2.1**
Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.

1 test question

$$\frac{10^{-2}}{10^{-4}} =$$

A 10^{-6}

B 10^{-2}

C 10^2

D 10^8

M02832

CAHSEE questions in this standard require students to understand the concept of negative exponents. One of the most powerful concepts in mathematics is that exponential notation can be extended to include new concepts. Negative exponents are an example of this kind of extension.

Sample Test Question
The correct answer is choice C. Students should understand that 10^{-2} is equivalent to

$\dfrac{1}{10^2}$ and $\dfrac{1}{10^{-4}}$ is equivalent to 10^4. Thus, it is possible to represent the problem as

$\dfrac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$, making the underlying concept more apparent. $\dfrac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10}$ can be reduced

by dividing it by $\dfrac{10 \cdot 10}{10 \cdot 10}$ to $10 \cdot 10$, which is equivalent to 10^2. Once the students master the

concept behind negative exponents, they understand that when dividing exponential expressions

with the same base, the exponents must be subtracted, so that $\dfrac{10^{-2}}{10^{-4}}$ is equivalent to $10^{(-2)-(-4)}$,

which is equivalent to 10^2.

Analysis of Distractors

Distractor A: added the exponents or made an error in the subtraction of $(-4)-(-2)$

Distractor B: did not apply the negative signs correctly in the same subtraction

      C: correct answer

Distractor D: multiplied -2 by -4

Strand   **Number Sense (NS)**

Standard   **7NS2.2**
**Add and subtract fractions by using
factoring to find common denominators.**

1 test question

Which fraction is equivalent to $\dfrac{5}{6} + \dfrac{7}{8}$?

A $\dfrac{35}{48}$

B $\dfrac{6}{7}$

C $\dfrac{20}{21}$

D $\dfrac{41}{24}$

M12713

The focus of this content standard is on the students' ability to add and subtract fractions with unlike denominators that share one or more factors. Students should be able to find the prime factorization of each denominator, then combine factors to determine the least common denominator. CAHSEE questions in this standard require students to perform addition and subtraction using equivalent fractions with common denominators. The algorithmic approach of this standard is associated with the requirement that common denominators be determined by factoring.

Sample Test Question
The sample requires students to find the needed common denominator for 6 and 8 using prime factors. The correct answer choice is D. Students should recognize that the prime factors for 6 are 2 and 3 and that the prime factors for 8 are 2 and 4. Since the common prime factor is 2, including the additional factors of 3 and 4 gives $2 \cdot 3 \cdot 4$ as the prime factors of the least common denominator. Students should then find equivalent fractions using the least common denominator and add the fractions.

Analysis of Distractors

Distractor A: multiplied the numerators and denominators together, respectively

Distractor B: added the numerators and denominators together, respectively and reduced the fraction to its lowest form

Distractor C: the numerators of the equivalent fractions expressed as a numerator and denominator

D: correct answer

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Strand **Number Sense (NS)**

Standard **7NS2.3**
Multiply, divide, and simplify rational numbers by using exponent rules.

1 test question

$$\left(3^8\right)^2 =$$

A 3^4

B 3^6

C 3^{10}

D 3^{16}

M02406

CAHSEE questions in this standard require students to select the appropriate rules for operations with exponents with common bases and perform accurate computations in simplifying rational numbers. Students should understand the following rules:
- adding exponents when multiplying numbers with common bases
- subtracting exponents when dividing numbers with common bases
- multiplying exponents when raising a number to a particular power

Questions may include those requiring multistep operations, such as the simplification of numerators and denominators with common factors.

Sample Test Question
The correct answer is choice D. In this instance, students should use the rule for multiplying exponents $\left[\left(a^b\right)^c = a^{bc}\right]$ to determine that $\left(3^8\right)^2 = 3^{16}$.

Analysis of Distractors

Distractor A: used division instead of multiplication for the exponents

Distractor B: used subtraction instead of multiplication for the exponents

Distractor C: used addition instead of multiplication for the exponents

D: correct answer

Strand **Number Sense (NS)**	**The square root of 150 is between—**
Standard **7NS2.4** **Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.**	**A** 10 and 11. **B** 11 and 12. **C** 12 and 13. **D** 13 and 14. M02666

1 test question

CAHSEE questions in this standard require students to demonstrate a conceptual understanding of powers and roots and their inverse relationship. The idea of mathematical inverse is a key precursor for algebraic reasoning, and students should understand that taking a root is the inverse operation of raising a number or expression to a power. For example, students should know that $\sqrt{9^2} = 9$. Students will not be required to calculate the square root for a number that is not a perfect square, but students should be able to approximate the value of the square root of an integer that is not a perfect square.

Sample Test Question
The correct answer is choice C. Students should recognize that they must first determine both the closest perfect square greater than the given integer and the closest perfect square less than the integer. This task may involve some trial-and-error multiplication along with the application of knowledge of squares and square roots. Since the number 150 is not a perfect square, students may recall or calculate that 144 is a perfect square and that 169 is the next perfect square. Since 144 is close to but less than 150 and 169 is close to but greater than 150, the square root of 150 must lie between those two perfect squares.

Analysis of Distractors
The distractors misplace 150 between other numbers and may be selected by students who do not understand the concept of square root or who may incorrectly calculate the square of one of the numbers.

Number Sense

Strand   **Number Sense (NS)**

Standard   **7NS2.5**
Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

1 test question

If $|x| = 3$, what is the value of x?

A -3 or 0

B -3 or 3

C 0 or 3

D -9 or 9

M02122

CAHSEE questions in this standard require students to demonstrate a conceptual understanding of absolute value and its meaning as represented on a number line. Relating absolute value to distance on the number line may help students understand the concept: Distance cannot be negative, but there will always be two numbers on the number line that are the same distance from zero. Questions may require students to find the absolute value after performing a basic computation.

Sample Test Question
The correct answer is choice B. Students should recognize that since the absolute value of a number is the distance on a number line from that number to zero in either direction, all

absolute values are positive numbers: $|x| = x$ and $|-x| = x$. In the test question, if $|x| = 3$,

then $x = -3$ or $x = 3$, since $|3| = 3$ and $|-3| = 3$.

Analysis of Distractors

Distractor A: equated the absolute value of 3 with 0 and a negative value of 3

          B: correct answer

Distractor C: incorrectly equated the absolute value of 3 with 0 and a value of 3

Distractor D: associated the absolute value of 3 with the square of 3